Bouquet: A Visualization Tool for Symmetry Sets and Vineyards

Erin Chambers[1], Christopher Fillmore[2], Shankha Shubhra Mukherjee[1], Rohit Roy[3], Elizabeth Stephenson[4], Mathijs Wintraecken[3]

EuroCG 2026 • Hagen, Germany

Try it out!

[1]University of Notre Dame
[2]Institute of Science and Technology Austria
[3]Inria Centre Université Côte d'Azur
[4]Orteliu

3 Questions

1. What?
2. Why?
3. How?

Preliminaries

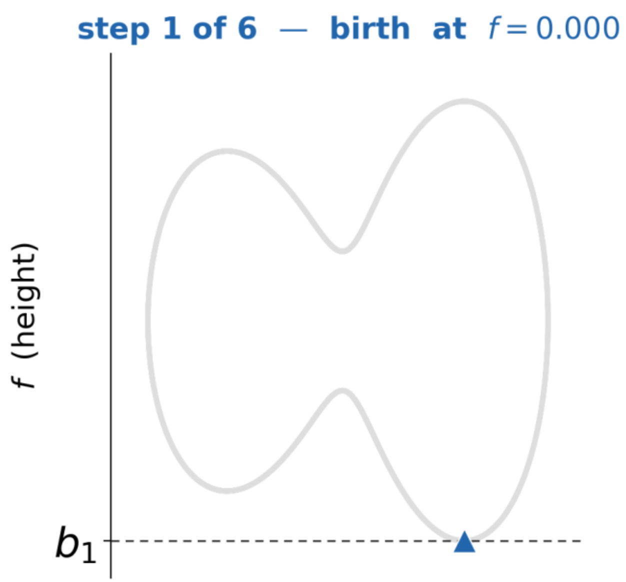
Persistence Diagrams

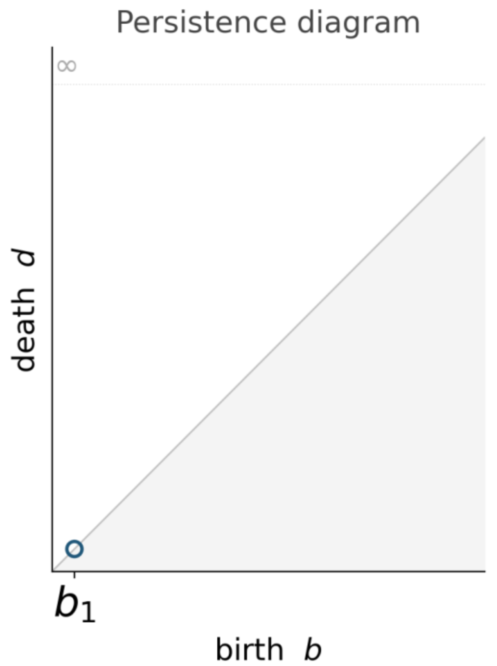
- Given a nested sequence of spaces (a ***filtration***), topological features appear and disappear as the space grows
- A persistence diagram records these **(birth, death)** pairs as points in $\mathbb{R}^2$, summarizing the "shape" of data across all scales

Preliminaries

Persistence Diagrams

- Given a nested sequence of spaces (a ***filtration***), topological features appear and disappear as the space grows
- A persistence diagram records these **(birth, death)** pairs as points in $\mathbb{R}^2$, summarizing the "shape" of data across all scales

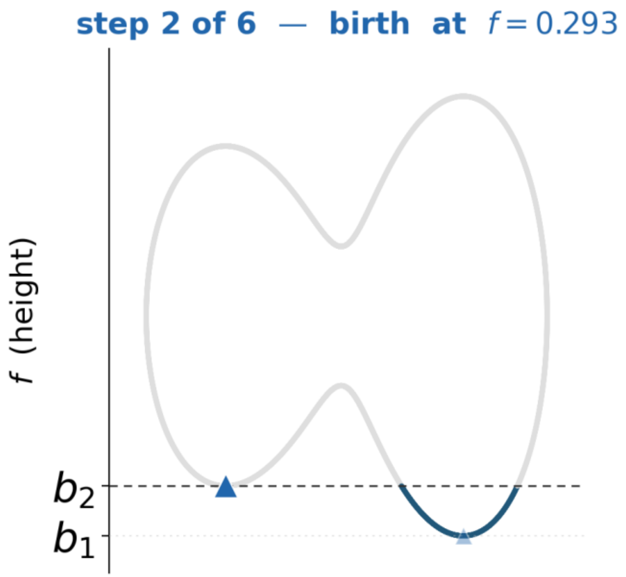


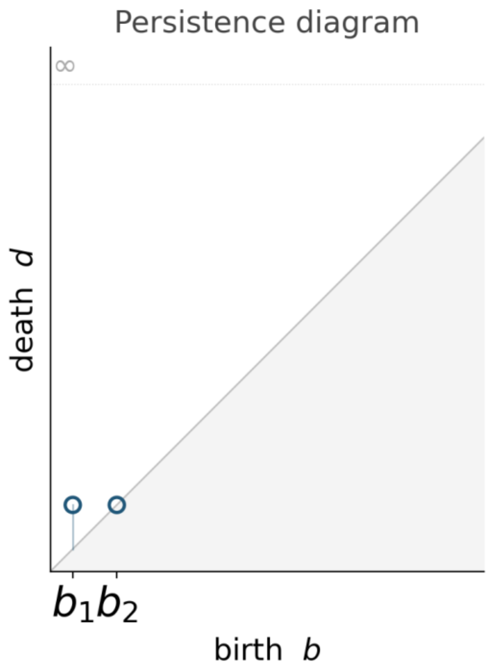


Preliminaries

Persistence Diagrams

- Given a nested sequence of spaces (a ***filtration***), topological features appear and disappear as the space grows
- A persistence diagram records these **(birth, death)** pairs as points in $\mathbb{R}^2$, summarizing the "shape" of data across all scales

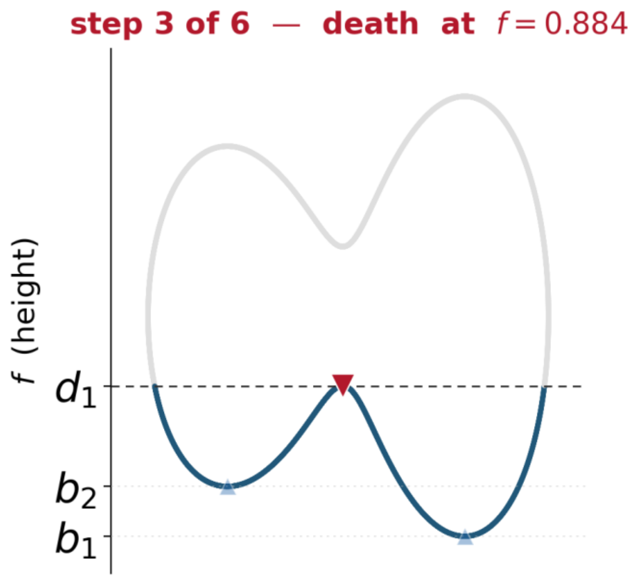


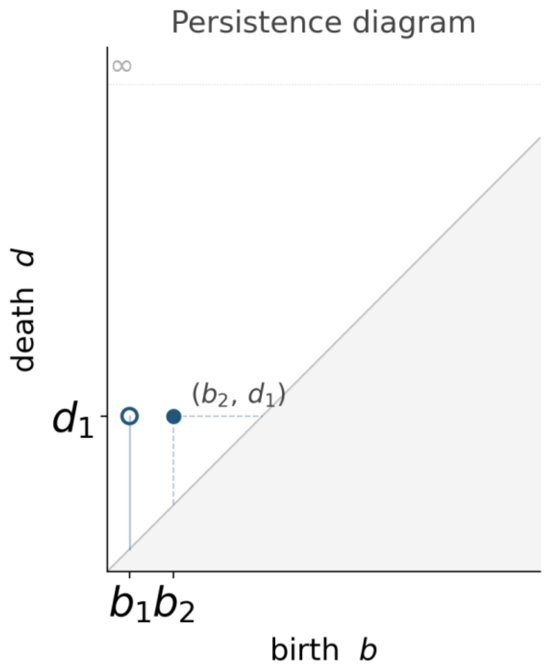


Preliminaries

Persistence Diagrams

- Given a nested sequence of spaces (a ***filtration***), topological features appear and disappear as the space grows
- A persistence diagram records these **(birth, death)** pairs as points in $\mathbb{R}^2$, summarizing the "shape" of data across all scales

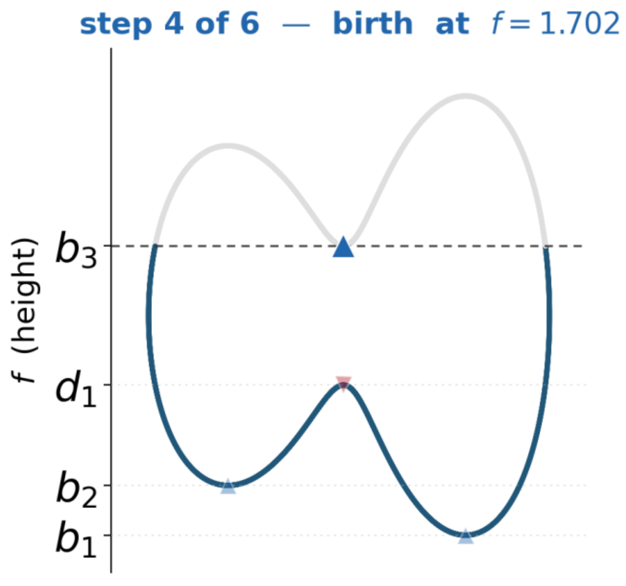


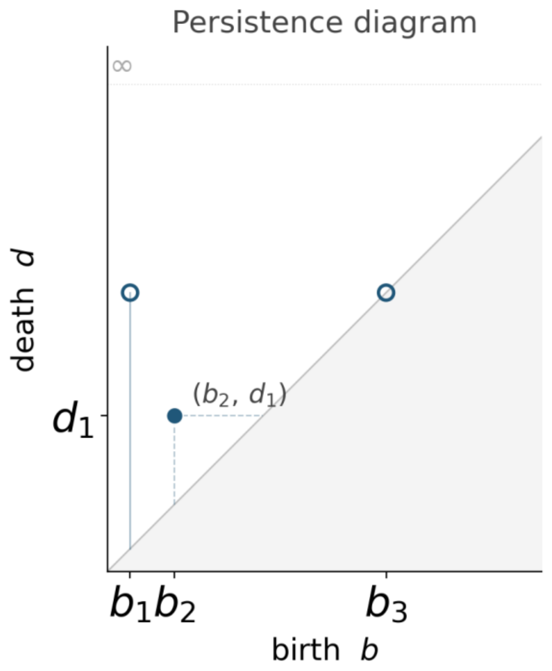


Preliminaries

Persistence Diagrams

- Given a nested sequence of spaces (a ***filtration***), topological features appear and disappear as the space grows
- A persistence diagram records these **(birth, death)** pairs as points in $\mathbb{R}^2$, summarizing the "shape" of data across all scales

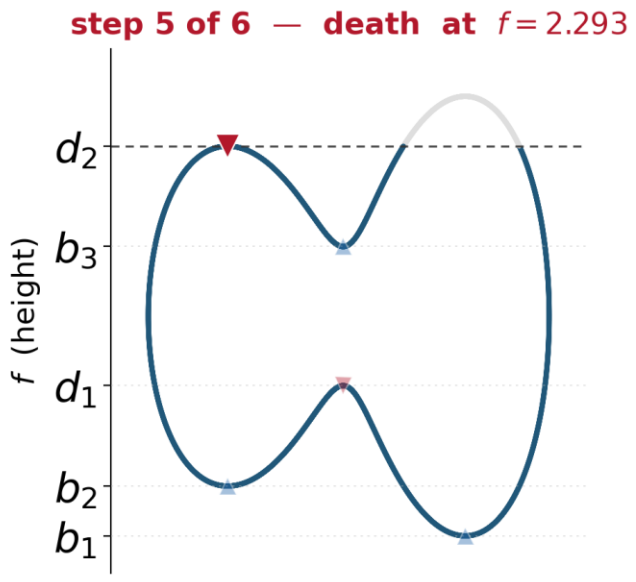


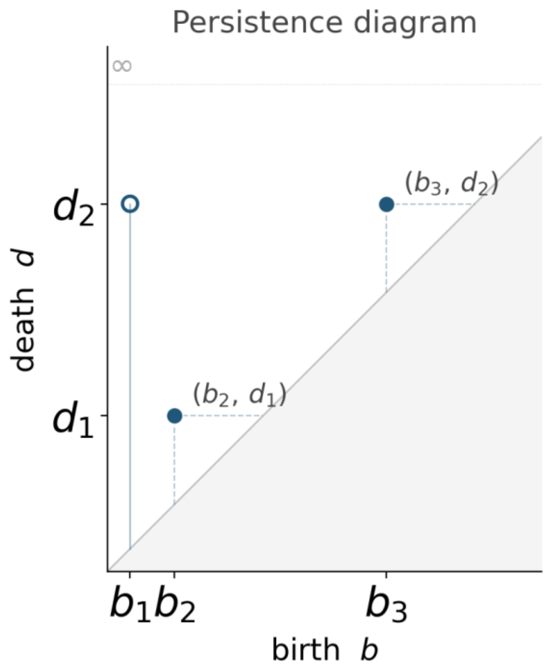


Preliminaries

Persistence Diagrams

- Given a nested sequence of spaces (a ***filtration***), topological features appear and disappear as the space grows
- A persistence diagram records these **(birth, death)** pairs as points in $\mathbb{R}^2$, summarizing the "shape" of data across all scales

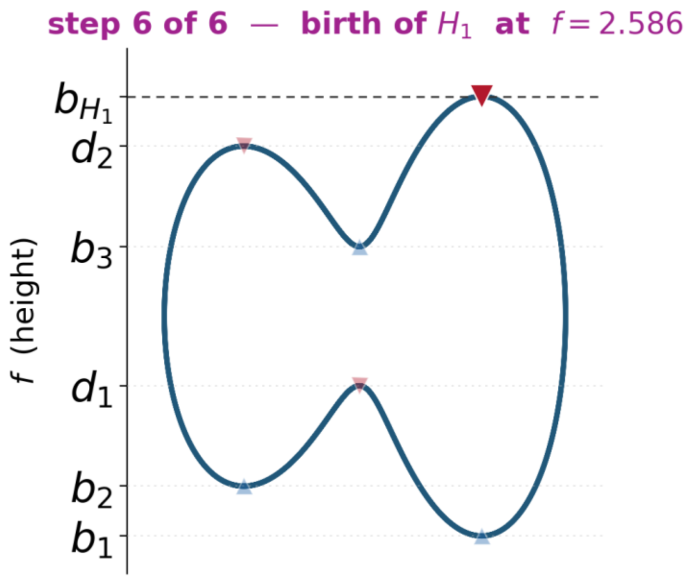


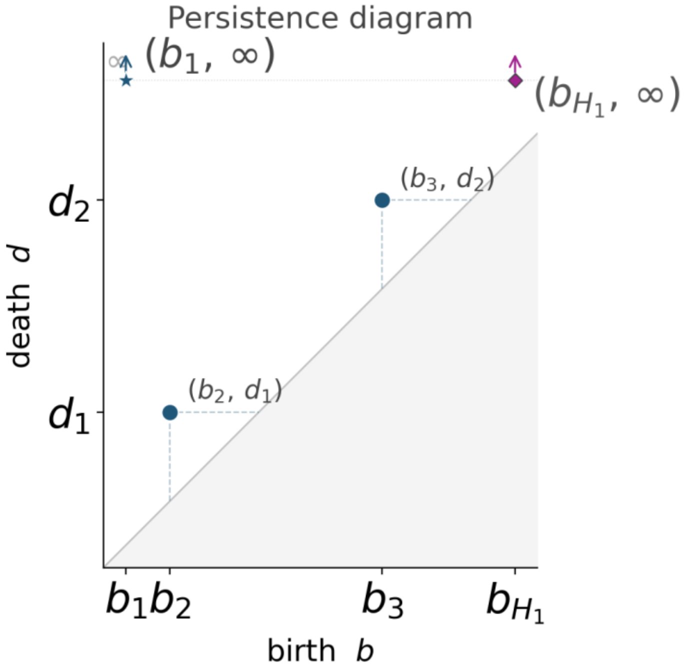


Preliminaries

Persistence Diagrams

- Given a nested sequence of spaces (a ***filtration***), topological features appear and disappear as the space grows
- A persistence diagram records these **(birth, death)** pairs as points in $\mathbb{R}^2$, summarizing the "shape" of data across all scales

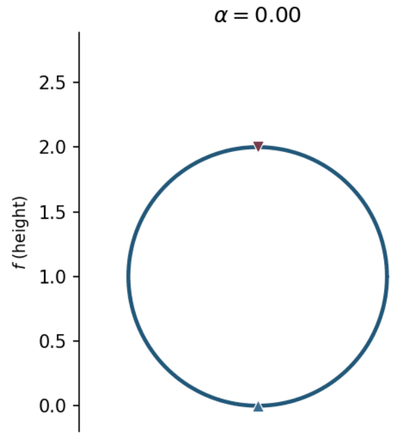




Preliminaries

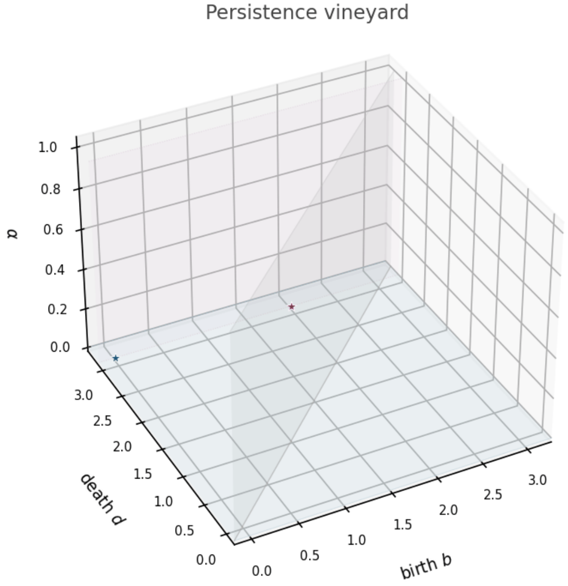
Persistence Diagrams

- Given a nested sequence of spaces (a ***filtration***), topological features appear and disappear as the space grows
- A persistence diagram records these **(birth, death)** pairs as points in $\mathbb{R}^2$, summarizing the "shape" of data across all scales

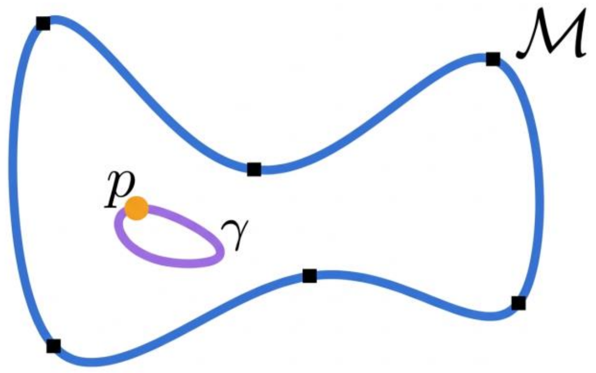


Vineyards

- A 1-parameter *family of filtrations* gives continuously changing persistence diagrams
- Stacking all diagrams together gives a **vineyard**
- Individual points trace out curves called **vines**



Preliminaries



Restricted Distance Function measures the distance from a point p to every point on a curve $\mathcal{M}$.

The persistence diagram associated with the restricted distance function is the **Radial Persistence Transform.**

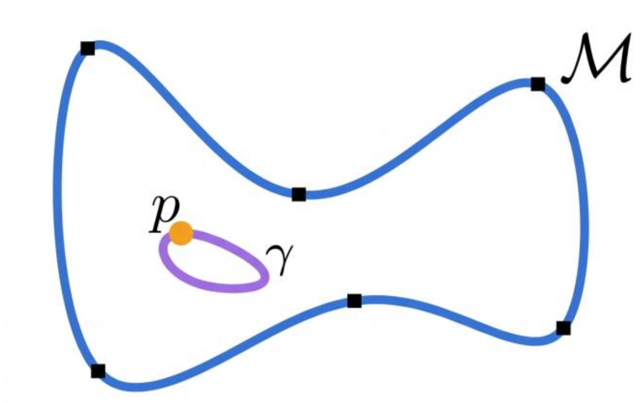
When $p = \gamma(t)$ traverses a **loop** ($\gamma \simeq S^1$), the persistence diagrams vary continuously, which give us the **vineyard**.

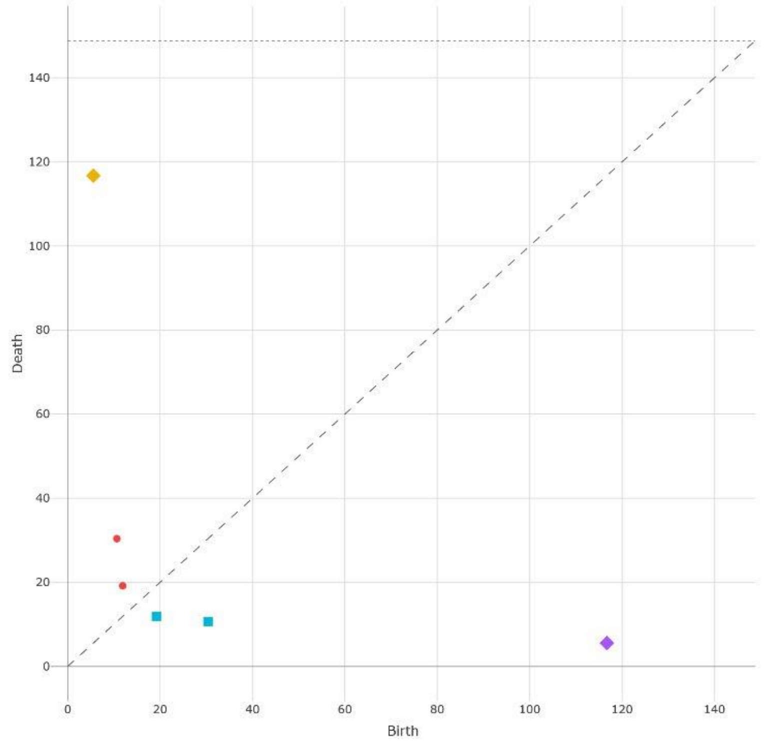
Preliminaries

Restricted Distance Function measures the distance from a point p to every point on a curve $\mathcal{M}$.

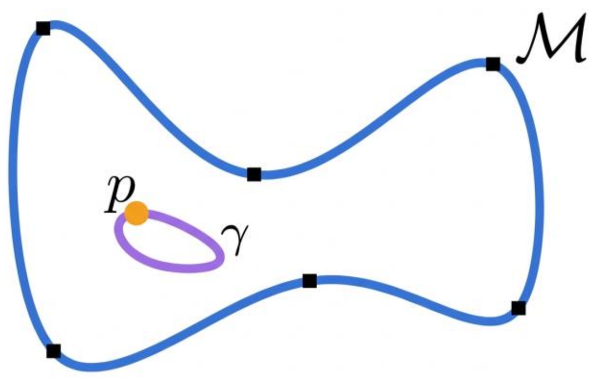
The persistence diagram associated with the restricted distance function is the **Radial Persistence Transform.**

When $p = \gamma(t)$ traverses a **loop** ($\gamma \simeq S^1$), the persistence diagrams vary continuously, which give us the **vineyard**.





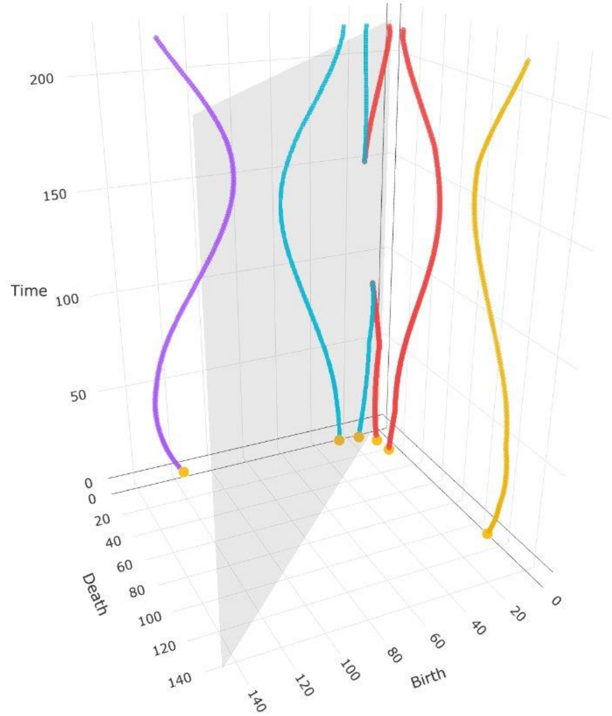
Preliminaries



Restricted Distance Function measures the distance from a point p to every point on a curve $\mathcal{M}$.

The persistence diagram associated with the restricted distance function is the **Radial Persistence Transform.**

When $p = \gamma(t)$ traverses a **loop** ($\gamma \simeq S^1$), the persistence diagrams vary continuously, which give us the **vineyard**.



Evolute (Focal Set)

Definition

The **evolute** is the locus of centers of curvature — the centers of "best-fit circles" at each point on $\mathcal{M}$.

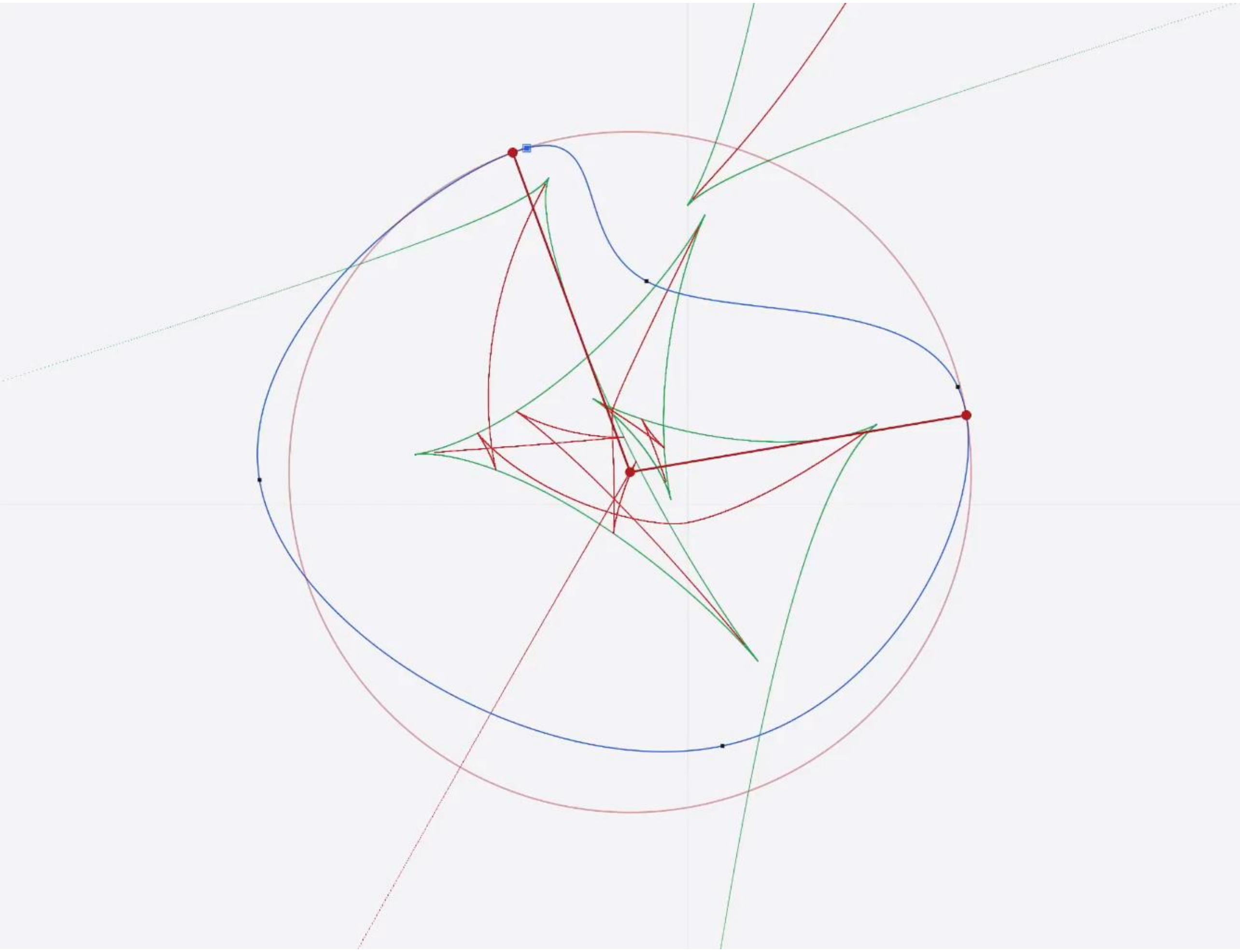
Symmetry Set

Definition

The **symmetry set** of a curve $\mathcal{M}$ is the set of all centers of circles that are tangent to $\mathcal{M}$ in at least two places.

Evolute + Symmetry Set
= **Bifurcation Set**

- *Topology of the distance function can change on crossing the set*
- *Can induce changes in the vineyard!*



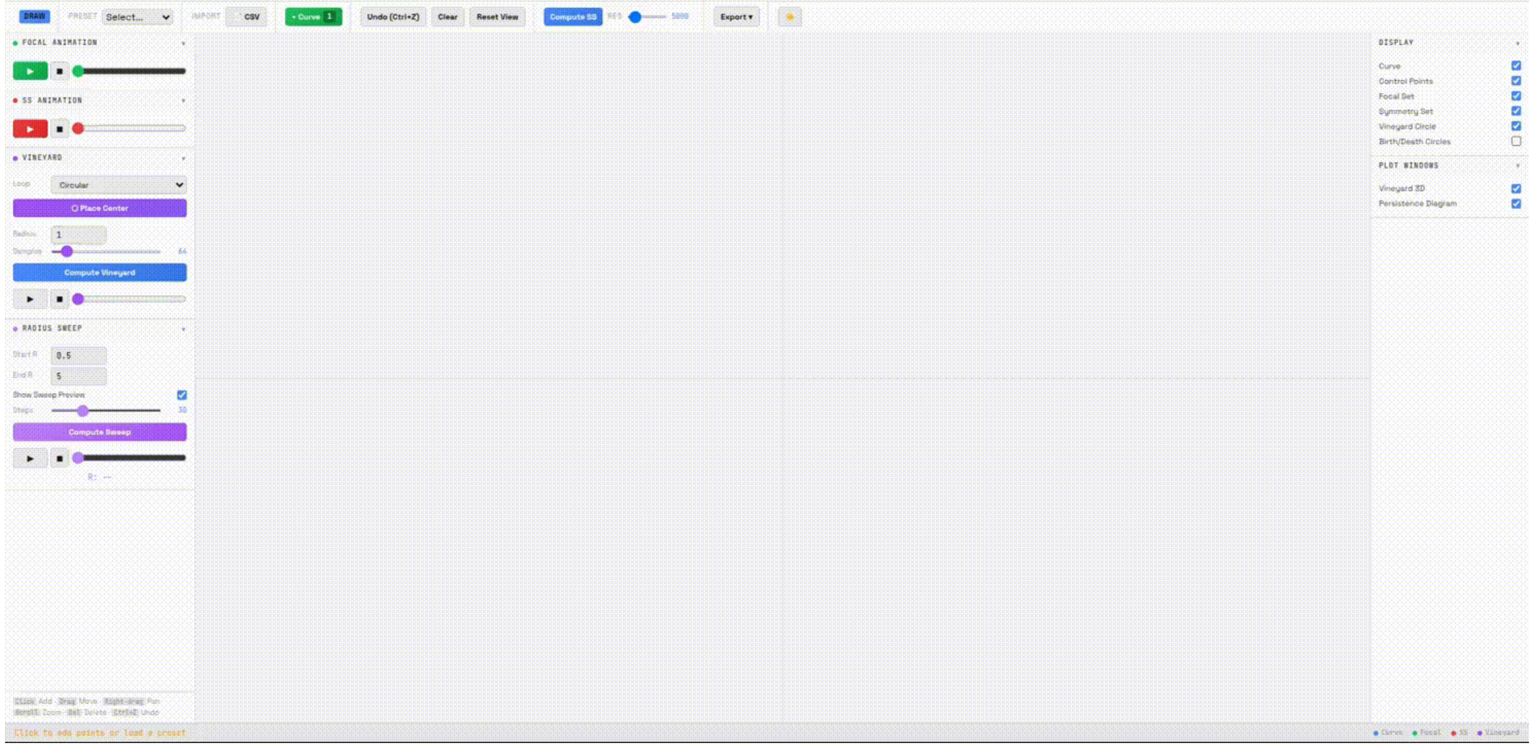
3 Questions

1. What?
2. Why?
3. How?

What is Bouquet2D?

Web-based tool that visualizes:

- Bifurcation set
- Vineyards of the radial persistence transform



Draw *any* curve $\mathcal{M}$ using splines and see how the bifurcation set and vineyard evolve!

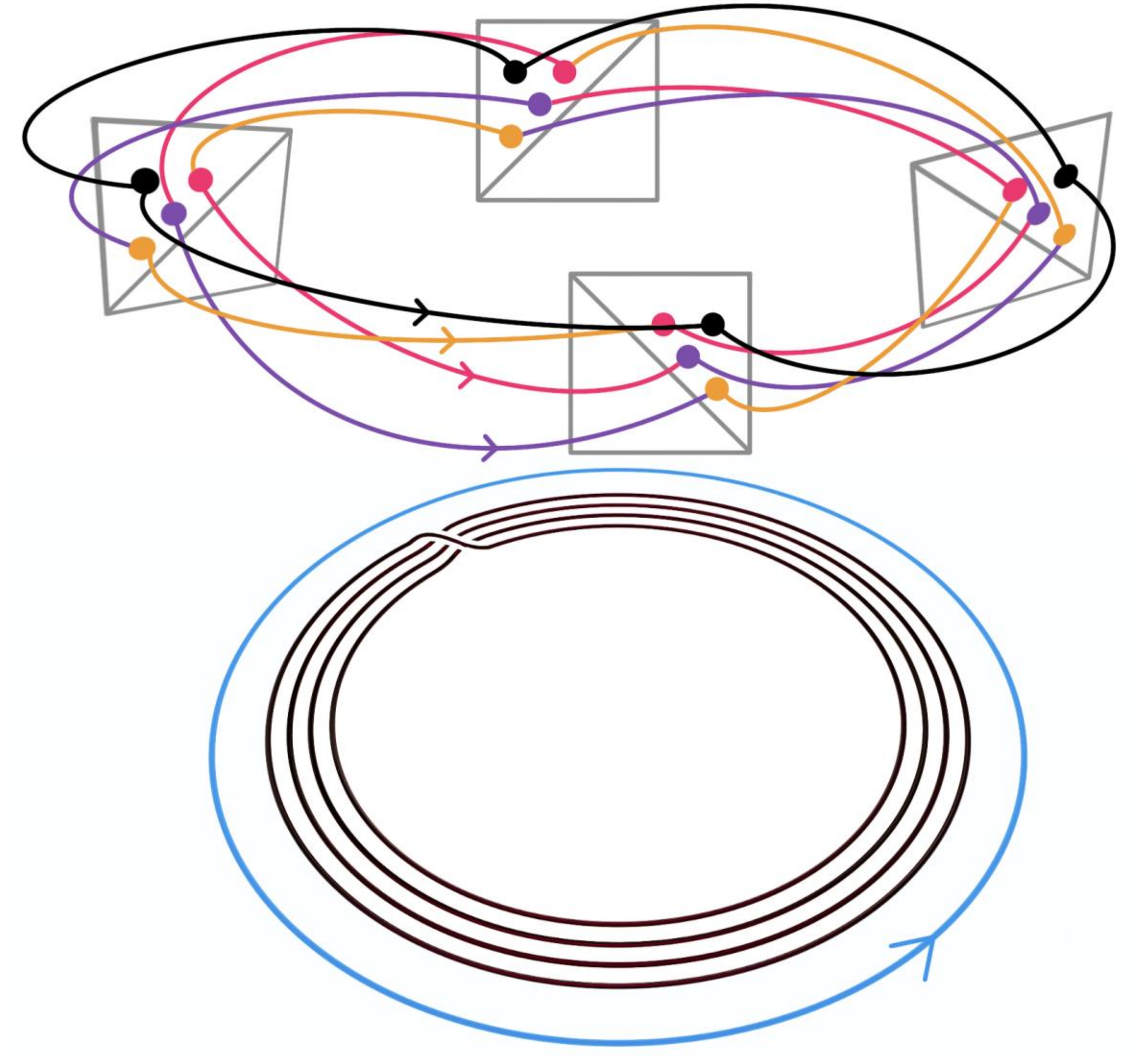
3 Questions

1. What?
2. Why?
3. How?

Monodromy

Vineyards have surprising complexity

- Follow a vine for one full loop around γ
- You might **not** return to the same persistence point!
- This is called **monodromy** (order k = number of rotations needed to return)



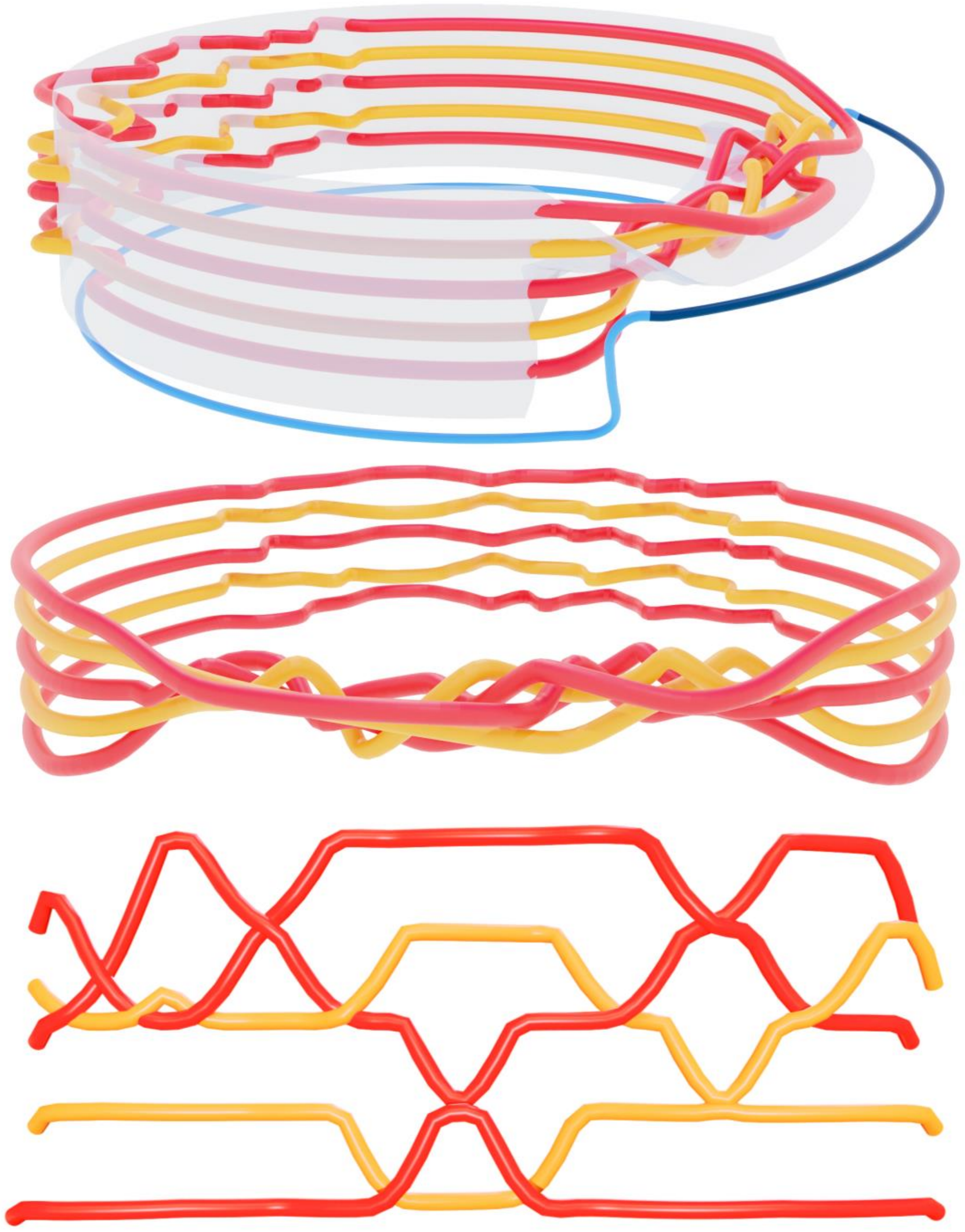
Monodromy

Vineyards have surprising complexity

- Follow a vine for one full loop around γ
- You might **not** return to the same persistence point!
- This is called **monodromy** (order k = number of rotations needed to return)

Recent results [Chambers et al., SODA 2026]

- Any order of monodromy can occur
- Vines can braid - even form knots!
- *Why is there monodromy at all? (Open question!)*



The Key Question

How is the topology of the bifurcation set related to the topology of the vineyard?

Monodromy, braiding, and knots in vineyards are determined by the geometry (singular structure) of the symmetry and focal sets.

Bouquet2D explores this connection interactively and visually.

3 Questions

1. What?
2. Why?
3. How?

A little demo!

Outlook

Current Status

- 2D tool is live
- Supports spline-based curve deformation in real time
- Handles arbitrary loops and multiple connected components

Future Work

- Extension to higher dimensions

Thank you!

bouquet2d.rohitroy.me

Bouquet2D: The Tool

Symmetry Set Computation (Giblin et al.)

- Detect bitangent circles via *zero-crossings*
- Compute centers via normal line intersections

Vineyard Computation

- Move a point along a user-defined loop
- Compute persistence at each step (Union-Find)
- Stack diagrams to build the vineyard in real time